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Opinion

Symbols and mental programs: a hypothesis about human singularity

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Natural language is often seen as the single factor that explains the cognitive singularity of the human species. Instead, we propose that humans possess multiple internal languages of thought, akin to computer languages, which encode and compress structures in various domains (mathematics, music, shape…). These languages rely on cortical circuits distinct from classical language areas. Each is characterized by: (i) the discretization of a domain using a small set of symbols, and (ii) their recursive composition into mental programs that encode nested repetitions with variations. In various tasks of elementary shape or sequence perception, minimum description length in the proposed languages captures human behavior and brain activity, whereas non-human primate data are captured by simpler nonsymbolic models. Our research argues in favor of discrete symbolic models of human thought.

Highlights

Accounting for human spatial memory requires the postulation of a mental language that can recursively compose primitives of number, space, and repetition with variations.

The same language accounts for the human perception of binary auditory sequences.

Minimum description length, rather than actual sequence length, predicts human working memory for auditory and visual sequences.

When perceiving geometric shapes, humans exhibit a strong geometric regularity effect, which is absent in non-human primates.

Multiple languages with similar computational principles but distinct, parallel brain circuits coexist in the human brain.

> [The universe] cannot be read until we have learnt the language and become familiar with the characters in which it is written. It is written in mathematical language, and the letters are triangles, circles and other geometrical figures, without which means it is humanly impossible to comprehend a single word.
> Galileo Galilei, *Il Saggiatore* (The assayer), 1623.

A universal human predilection for symbolic structures

The Lascaux cave, south of France, is famous for its spectacular depictions of aurochs, horses, and deer, from over 18 000 years ago. A lesser-known fact, however, is that prehistoric humans also left many nonfigurative **signs** (see Glossary) such as rectangles, series of dots, etc. (Figure 1). Abstract geometrical patterns are omnipresent in human productions throughout the globe and predate figurative art by hundreds of thousands of years. For instance, early *Homo sapiens* left a network of parallel lines and equilateral lines in ocher at Blombos Cave, South Africa, ~70 000–100 000 years ago [1], and *Homo erectus* drew parallel lines and zigzags on a shell in Java ~540 000 years ago [2]. Even after years of human contact, non-human primates, when given pencils, never produce such structured geometrical shapes and drawings [3]. Thus, humans are different, even in domains such as drawing and geometry that do not involve communicative **language**. We refer to this observation using the term ‘human cognitive singularity’, the word singularity being used here in its standard meaning (the condition of being singular) as well as its mathematical sense (a point of sudden change). Hominization was certainly a singularity in biological evolution, so much so that it opened up a new geological age (the Anthropocene). Even if evolution works by small continuous change (and sometimes it does not [4]), it led to a drastic cognitive change in humans.

Many hypotheses have been proposed for human singularity, including a special competence for analogy, theory of mind, teaching, cultural memory, or interindividual communication [5–9]. Here,

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https://doi.org/10.1016/j.tics.2022.06.010

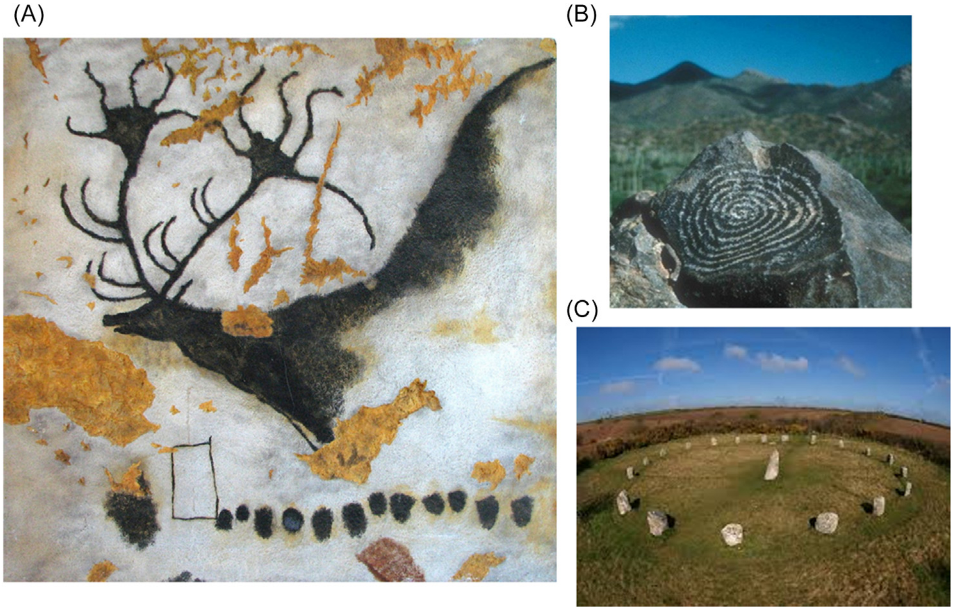

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Figure 1. Ubiquity of geometrical structures in human cultures. Examples of small- and large-scale geometric drawings and constructions. (A) Geometrical shapes below the painting of a *Megaloceros* in Lascaux, France, ~18 000 years old. (B) Spiral stone engraving on Signal Hill in Saguaro National Park, Arizona, from 550 to 1550 years ago. (C) Boscawen-Ûn's Bronze Age elliptical cromlech in Cornwall.

we propose that the deceivingly simple ability to draw a zigzag points to a deep cognitive difference, which may have impacted all of the aforementioned competences. Why are humans the only species to have developed not only spoken and written languages, but also the languages of science, music, mathematics, visual patterns…? We argue that behind each of these domains of competence lies a specifically human mode of mental representation: discrete **symbols** and their composition in a **recursive language** allow our species to build arbitrarily complex mental structures out of a very small set of initial primitives. Charles Sanders Peirce famously distinguished three types of signs: **icon**, **index**, and symbol (see [10–12]). We suggest that humans owe their singularity to symbols.

Experimentally, using highly simplified perceptual paradigms, we found that even the simplest aspects of human perception, such as how we see a square, perceive a binary rhythm, or draw a zig-zag, are different in humans, in a way that can only be explained by appealing to such discrete and compositional mental representations. Of course, other animals perceive the difference between a square and a circle, but if our hypothesis is correct, they do so using only classical visual recognition mechanisms, whereas humans also engage a logical, recursive mode of representation akin to a programing language.

The present proposal is closely related to a long tradition of theorizing that placed a key emphasis on the central role of recursive tree structures in human language and cognition [13–16] (Box 1). Tecumseh Fitch, in particular, stressed the specifically human propensity to construct tree structures out of any data, which he dubs 'dendrophilia' [14]: 'Humans have a multi-domain capacity and proclivity to infer tree structures from strings, to a degree that is difficult or impossible for most non-human animal species' [14]. Our results disagree with this statement on only one point: there does not seem to be a single multi-domain faculty or core brain circuit for syntactic structures.

Glossary

Sign: according to Ferdinand de Saussure's classic definition, a sign is a pairing, often arbitrary and conventional, between a 'signifier' (the physical form of the sign) and a 'signified' (the object or concept that is being referred to). Confusingly, the word sign is sometimes used to refer solely to the signifier (e.g., a road sign). Following Charles Sanders Peirce, the forms of signs can be sorted into three categories: icon, index, and symbol.

Icon: a signifier that bears a physical resemblance to its signified object or concept, for instance, a schematic drawing of a plane ✈ as a sign for an airport.

Index: a signifier that bears an arbitrary but regular relation of spatial or temporal proximity to its referent and can therefore 'point to it', evoking it by association. For instance, a bell that tells an animal that food is coming. Animal communication, whether spontaneous or trained in the laboratory, relies on indexical relations.

Symbol: a signifier that can enter into syntactic relations with other symbols to form a system or language. A symbol does not exist in isolation but participates in a symbol system in which the symbolic relationships parallel some of the relationships between the corresponding concepts. For instance, in the expression '2 + 1 = 3', a symbolic manipulation predicts the result of adding one item to a set of two. Symbols can be external (e.g., the physical mark '2' on this page) but also internal to the brain (e.g., the neural assembly that physically represents number 2 in your brain and can combine with others to form mental expressions such as '2 + 1 = 3').

Language: unless otherwise stated, we use the term 'language' in the technical sense of formal language theory: a system of rules that compose elementary symbols to form arbitrarily complex expressions.

Program: a sequence of symbols that act as primitive instructions in a language that a machine (including the brain) can process and convert into a series of operations. Here, we allow for mental programs, that is, internal expressions that contain a set of instructions and can be unfolded into a series of mental processes (e.g., for drawing a square).

Recursive language: a language that allows instructions to be nested inside

Box 1. Symbols and languages: a hypothesis with a rich history in cognitive science

The hypothesis that human thought rests on a language of symbols and grammatical rules has a long history (e.g., Augustine, Ockham, Descartes, Boole, Peirce…) [110]. In 1936, Alan Turing proposed that the operations in a mathematician's mind could be captured, in first approximation, by a Turing machine, a simple device that writes and reads symbols from a tape according to conditional rules forming programs [111,112]. In 1951, Karl Lashley gave many arguments suggesting that the human competence for sequence processing goes beyond simple associative chains and instead involves nested plans [113]. In 1957, Noam Chomsky formulated his influential argument against the then-dominant Skinnerian view of language [114]: sentences, with their potentially unlimited embeddings of phrases within phrases, were not mere associative chains of words, but tree structures emerging from the recursive application of rules forming a generative grammar.

In parallel, a lesser known line of research developed equally precise proposals concerning the human representation of sequential and visual rules. In 1967, George Miller [115] developed his project Grammarama, a 'program of laboratory experiments to investigate how people learn the grammatical rules underlying artificial languages'. Herbert Simon designed a computer-like language that captured human intuitions of regularities in letter sequences [56,116] and postulated that whenever humans hear or see a sequence, they infer and store in memory a short program capable of reproducing it. Leeuwenberg proposed a highly detailed language capable of describing any regular sequential pattern [117] as well as 2D and 3D shapes [54]. Leeuwenberg argued that the human perceptual system constantly seeks the most compact interpretation of stimuli in this internal language, the least 'structural information load'. The resulting 'structural information theory' compared favorably with previous structuralist or Gestalt approaches to perception (as reviewed by [118]) and received further experimental support by Frank Restle (e.g., [55,119–121]) as well as extensions to more complex composite shapes [53,122–124].

The generative grammar approach had a broad influence on theories of the grammar of rhythm and music [57,58,91,92][i], mathematics [23,125], concepts [93,126–130], or theory of mind [131,132]. It is integral to contemporary Bayesian theories of rational inductive inference, which view human learning as a search for the most probable internal representation of incoming data in a probabilistic language of thought or context free grammar [127,129,130,133–136]. Recent simulations suggest that a single Bayesian selection algorithm may learn many different languages and data structures [93,136].

The proposal that symbols and grammars are unique to humans also has a long history [10,11]. Hauser, Chomsky, and Fitch [13] famously proposed that the capacity to form recursive representations is absent in other animals and lies at the core of the human language faculty. The proposal was later extended to suggest that a competence for the mental representation and manipulation of nested tree structures, called dendrophilia [14], universal generative faculty [137], or recursive mental programs [138,139], underlies the singularity of the human mind in all cognitive domains [15].

Instead, evidence supports the existence of multiple parallel brain circuits hosting languages with partially distinct programing styles and domain-specific primitives (Figure 2). A syntactic core exists for spoken and written language [17–19], but the brain areas it mobilizes are distinct from those supporting the languages of music, mathematics, or shape. As we shall see, those languages share several properties on an abstract level (composition and **data compression** by recursive combination of discrete symbols), but also differ in others. For instance, the languages that capture geometrical and binary sequences emphasize repetition and symmetry, whereas many linguists have emphasized how natural languages avoid repetition and rely on antisymmetry [20,21]. Thus, the evolutionary changes responsible for hominization may have allowed multiple brain circuits to represent recursive structures.

Later, we first review three different domains in which we found evidence that humans deploy mental **programs** in a language of thought: spatial sequences, auditory sequences of sounds, and visual shapes. In each case, we provide behavioral evidence in favor of the language approach, then describe the underlying neural substrate, and, when available, the evidence for human singularity. In a final section, we synthesize what those languages share and how they lead to a generic proposal about human singularity.

A geometric language for spatial sequences

All neuropsychologists are familiar with the Corsi block tapping task, which evaluates spatial working memory. On each trial, the experimenter taps a certain number of blocks spread in

each other. For instance, in the language we propose for geometric and sequential patterns, the *Repeat* instruction can repeat a specific symbol [$\mathit{Repeat}_4(A) = AAAA$] but also another *Repeat* instruction: $\mathit{Repeat}_2[\mathit{Repeat}_2(A)\ \mathit{Repeat}_2(B)] = AABBAABB$.

Data compression: the search for a compact description for data that takes up less space in human or computer memory. There are multiple compression algorithms, such Lempel-Ziv-Welch (LZW), which differ in the regularities they can detect and, therefore, in the compression rate they attain.

Minimum description length (MDL): the size of the data once it is compressed to the shortest possible description in a given language or compression scheme.

Kolmogorov complexity: a mathematical concept, due to Solomonoff, Kolmogorov, and Chaitin, roughly equivalent to minimum description length, but for universal Turing machines. The Kolmogorov complexity of a string is the length of the shortest program that generates it. Kolmogorov complexity is uncomputable (i.e., it is impossible to write a program that computes the Kolmogorov complexity for any arbitrary string).

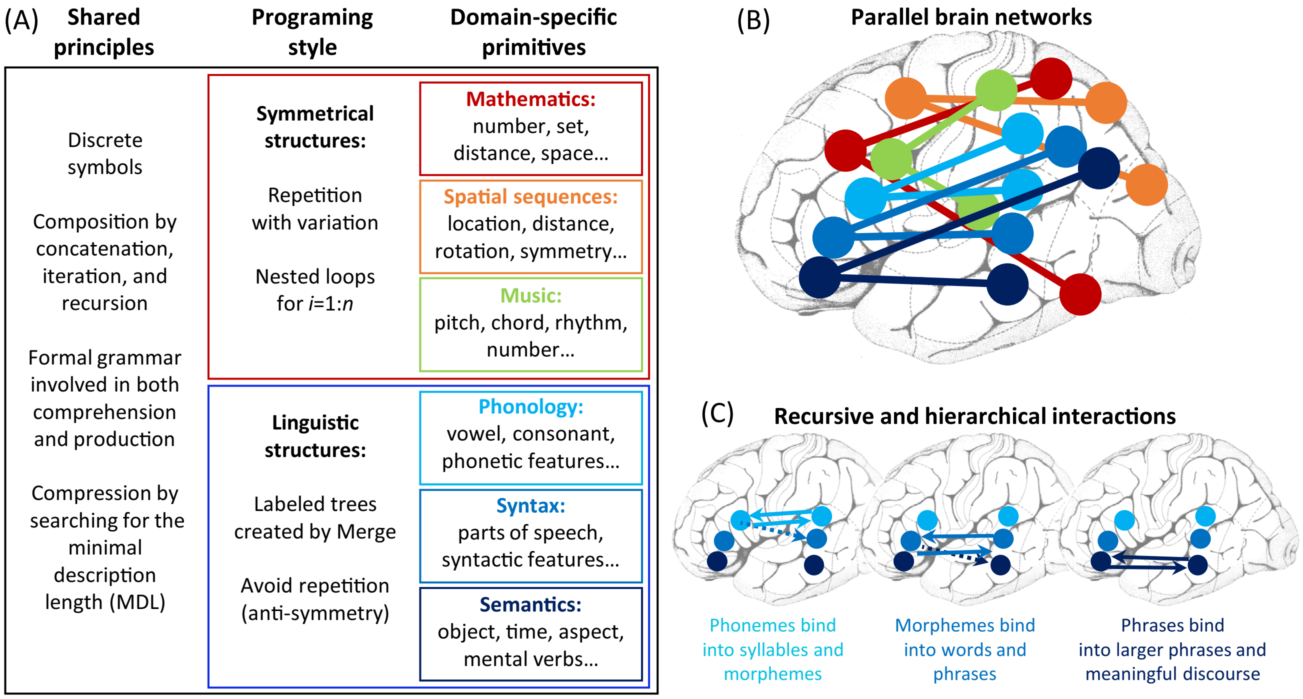

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Figure 2. Main hypotheses of the present proposal. (A) Multiple mental languages, all based on symbols and recursive mental programs. Various domains of human cognition rest upon several distinct internal languages, each capable of compressing different types of inputs. Those languages share the same design principles, but differ in their primitives. Two broad styles may be distinguished: one based on the capacity to detect repetition with variation, thus appropriate for encoding symmetrical patterns and mathematical structures; and another based on asymmetrical Merge, appropriate for encoding the structures of communicative language at multiple levels (this part of the figure is inspired by a previous proposal by Peter Hagoort [19]). Distinct languages emerge once these general instructions are combined with domain-specific primitives. (B) Multiple parallel cortical circuits. The proposed languages do not rely on a single localized brain area for recursion, but on multiple parallel brain networks with primitives in temporal and parietal cortex and control structures in prefrontal cortex. For simplicity, only a left hemisphere is shown, but the postulated brain circuits are generally bilateral. (C) Interactions within and between languages. The mental expressions formed in one language become available as primitives for the same or for another language, thus allowing for the formation of complex recursive and hierarchical thoughts (bottom right).

front of the participant, who has to reproduce the sequence in the correct order. In this task, working memory capacity typically does not exceed five or six locations, but this is true only for unstructured sequences. Whenever a spatial regularity is present, working memory is facilitated [22,23].

To systematically explore what factors determine the perceived regularity of a spatial sequence, in one study [23] children and adults were presented with sequences that traced the vertices of an octagon in various orders. All sequences therefore had the same length of eight, which exceeds typical working memory capacity. Indeed, for unstructured sequences, error rate exceeded 50%. However, as soon as the sequence comprised geometrical structures such as arcs, zig-zags, squares, or rectangles, performance was much better, not only in reproducing the sequence from memory, but also in anticipating upcoming locations [23,24]. Indeed, anticipations provided a rare case of zero-shot learning: the mere presentation of the first three to five locations sufficed to anticipate the rest without ever seeing the pattern in full. This was true, not only of educated adults, but also of preschoolers and Amazon Mundurucu speakers with little or no formal education.

What sort of regularities did they use? Three types: rotations (turning around the center of the octagon), symmetries (around one of the four axes of the octagon), and repetitions (possibly with variations). Those instructions could be concatenated or nested inside each other to create 'mental programs' in a computer-like language. A zig-zag, for instance, involved four repetitions of: (i) finding the next point on a side of the octagon, and (ii) applying a symmetry operation (Figure 3). A square was encoded as four repetitions of turning by 90° around the octagon and, therefore, a sequence of two squares was encoded as a twofold repetition of this fourfold repetition. The proposed language of geometry could encode any spatial sequence on the octagon as a recursively embedded series of such instructions (essentially, repetitions of repetitions).

Once this language was properly formalized, participants' working memory and anticipations could be accurately predicted. The key predictor was the complexity of the mental program needed to represent the sequence [i.e., its **minimum description length (MDL)**] (Figure 3). The idea is simple: in the absence of a language, a sequence of eight locations requires eight working memory slots. The language of geometry, however, allows compression of the sequence, using its regularities to render its memory representation more compact. Instead of listing the eight locations of a zig-zag, it is more efficient to state: 'repeat four times a symmetry operation, while moving the starting point by +1 along the octagon'.

The concept of MDL, also called 'the simplicity principle', has been heralded as a fundamental unifying principle for psychological science [25–27]. Indeed, MDL predicts human performance in many other domains, from digit span to concept learning [26–32]. The concept originates in algorithmic information theory [33,34], where a mathematical quantity called **Kolmogorov complexity** is the length of the shortest program that can produce a certain output. While Kolmogorov complexity, *sensu stricto*, applies only to universal Turing machines (where it is defined up to a constant), MDL applies to a fixed language and can be computed for small languages by simply enumerating their programs up to a given size [34]. In the present case, the length of those programs predicted how hard it was to remember the corresponding sequences.

We used brain imaging to garner direct evidence in support of the postulated language of thought and identify the underlying neural circuits. In fMRI [24], to avoid difficulty confounds, participants were merely presented with a spatial sequence and asked to follow it with their gaze. Importantly, even this elementary behavior was influenced by MDL, thus confirming a human proclivity to automatically encode linear sequences as recursive tree structures (Fitch's dendrophilia). The smaller the MDL, the more the eyes anticipated upcoming locations and landed there in advance of their actual appearance [Figure 3C; a similar result was obtained in magnetoencephalography (MEG)] [35]. Furthermore, MDL modulated the activity of a large set of bilateral dorsal occipito-parietal, prefrontal, and caudate regions (Figure 3D). Importantly, activation did not just increase with MDL, but decreased when sequences exceeded a critical level of MDL, indicating that no compact program could be found (for a similar result with verbal descriptions, see [36]). Thus, the activity of these regions reflected the structure of the postulated geometrical language code.

While fMRI signals pooled over the entire sequence, MEG provided direct evidence for the proposed internal code during the sequence itself [35]. To avoid eye movements, we asked subjects to fixate centrally while viewing a repeating spatial sequence, click when they thought that they had memorized it (encoding), and then detect an occasional deviant location (intruder detection). Behaviorally, both encoding time and intruder detection were again determined by MDL. Crucially, MEG signals contained direct information about the postulated numerical and

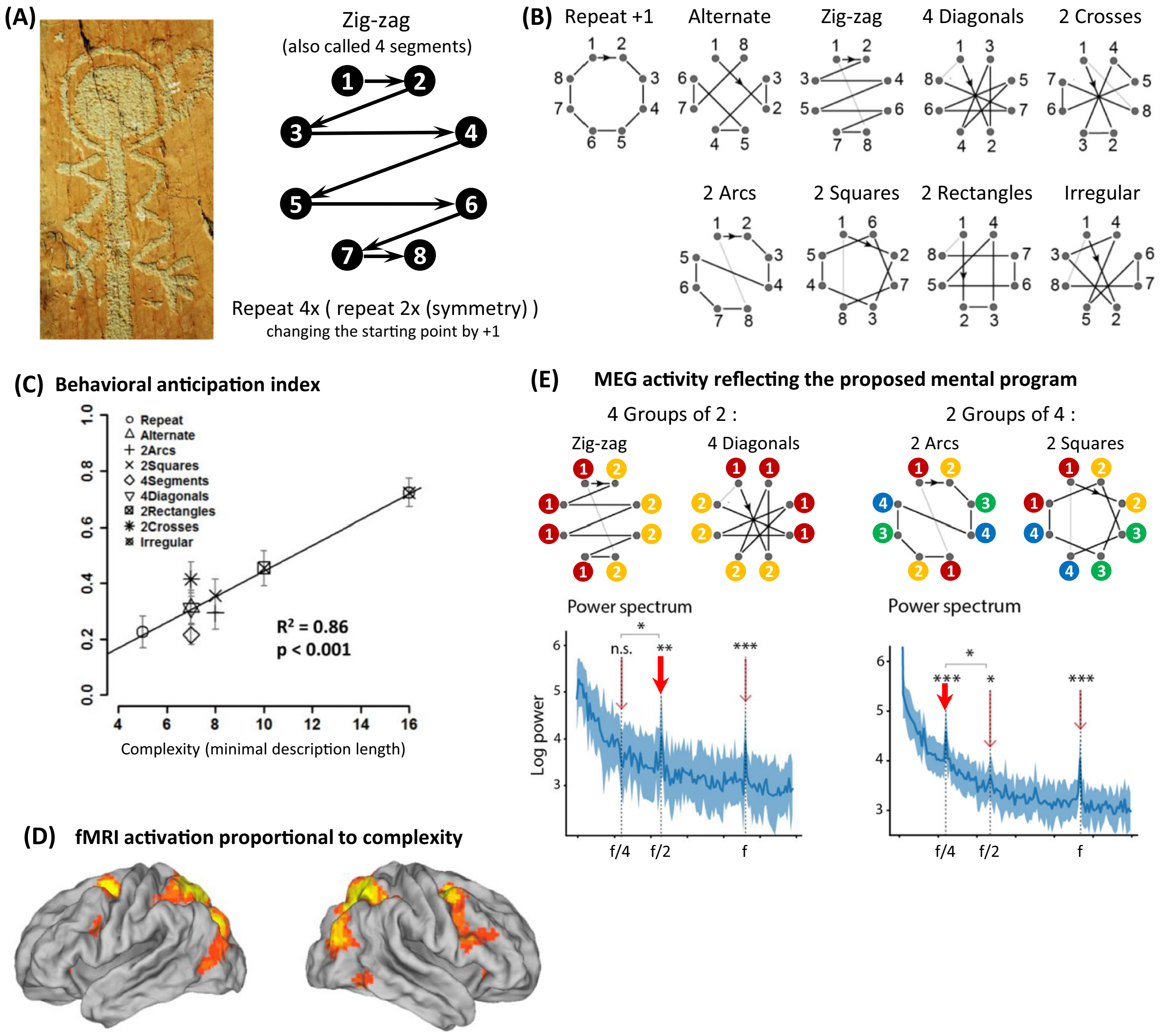

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Figure 3. A language for the human perception of visuospatial sequences. (A) Example of a spatial sequence, the zig-zag, here used to depict the arms of the 'God of storms' engraved ~3300–1800 BCE by the culture of Mount Bego (south of France). Amalric *et al.* [23] propose that the human production and perception of such spatial sequences requires a 'language of geometry' comprising nested repetitions of geometric operations (rotations and symmetries), with variations in starting point. (B) The proposed language was tested by presenting sequences of eight consecutive spatial locations on an octagon in behavioral and brain-imaging tests [23,24,35]. Here, the sequences are ordered by their predicted complexity [i.e. their minimum description length (MDL) in the proposed language]. (C) Behavioral evidence: the human error rate in storing the sequence in memory [23], anticipating the following items [23,24], and detecting outliers [35] is monotonically related to MDL; here the graph indicates the percentage of anticipatory eye movements [24]. (D) fMRI evidence: during eye tracking, fMRI activity is proportional to MDL, a vast bilateral occipito-parietal and dorsal premotor-prefrontal network, distinct from classical left-hemispheric language areas. (E) Magnetoencephalography (MEG) evidence. MEG signals contain decodable evidence about the structure of the proposed language, including the presence of primitives of arithmetic and geometry [35]. Furthermore, the predicted grouping structure (by groups of two or by groups of four) of sequences is directly reflected in the power spectrum of the decoded brain activity. Asterisks indicate the significance of a *t*-test for a difference in log power at the probed frequency relative to the average of the four neighboring frequencies: $^{*}P<0.05$; $^{**}P<0.01$; $^{***}P<0.001$.

geometrical primitives. They allowed to decode, for instance, whether a given transition between two locations was encoded as a rotation or a symmetry, even when the transition was physically identical, and its preferred encoding was solely determined by sequence context. MEG signals also allowed the ordinal position of items within a subsequence to be decoded. For instance, when tracing the four successive corners of a square, MEG signals contained numerical codes for the numbers one to four, suggesting that participants encoded the sequence using the mental equivalent of a ‘for i = 1:4’ loop.

Finally, in agreement with dendrophilia, MEG confirmed that, once it is mentally compressed in human working memory, a spatial sequence is no longer a flat linear structure, but consists of nested groupings. The theory predicts how the size of those groupings varies with the sequence: two squares consist of two groups of four, while a zig-zag consists of four groups of two, and an irregular sequence has no groupings at all. MEG supported this prediction: for each sequence, the spectrum of the decoded ordinal information showed peaks at frequencies corresponding to the predicted groupings (Figure 3E). This grouping signature, similar to the language domain [18,37,38], indicates that humans parse spatial sequences into nested constituent structures or phrases.

In summary, explaining human memory for spatial sequences requires a language of geometry. Crucially, fMRI showed that this language does not rely on classical language areas of the human brain, such as the inferior frontal gyrus (Broca’s area) or the superior temporal sulcus. Such areas, identified using a subject-specific localizer, were inactive or even deactivated during geometric language processing [24]. The activation induced by geometric sequences came close to Broca’s area in the dorsal part of bilateral Brodmann area 44d, but this site was not activated by sentences. Instead, the active areas overlapped significantly with those for mental arithmetic [39] and higher mathematics [40]. Those results fit with prior evidence that the language of mathematics can be anatomically and functionally dissociated from communicative spoken or written language; in normal subjects [40–42] and in brain-lesioned patients: agrammatic aphasics may still do algebra [43,44]. As stated by Galileo in the introductory quote, mathematics is a language whose symbols are numerical and geometrical rather than verbal.

A proto-musical language for binary auditory sequences

We next wondered if the notion of nested repetition could be generalized to the domain of auditory sequences. We restricted ourselves to binary sequences using only two sounds (call them A and B). In this case, the language becomes very simple: it merely specifies whether to stay with the same item, as in the minimal sequence AA, or to switch to the other, as in AB (where the items A and B could be sounds, locations, etc.). Nested repetitions of those primitives generate long yet compressible sequences such AABBABABAABBABAB (‘the repetition of a sequence formed by two pairs and four alternations’). Figure 4 shows a variety of such sequences, all of length 16, ranked according to predicted complexity (MDL).

Once transposed in this manner, the formal structure of our language of geometry predicted working memory for binary sound sequences [45]. We tested participants’ memory by habituating them to a given auditory sequence, then asking them to detect occasional deviants. As with geometry, response time and error rate in this intruder task were linearly predicted by MDL. This was true whether the deviants were transpositions (A instead of B) or much easier super-deviants (an unexpected sound C): the larger the MDL, the heavier the mental load and hence the lower the capacity to react to an unexpected sound. Explicit model comparison showed that MDL in the proposed language provided a better fit than several other competing measures such as transition probability, chunk complexity, entropy, subsymmetries, Lempel-Ziv compression, change complexity, or algorithmic complexity (see [45] for definitions).

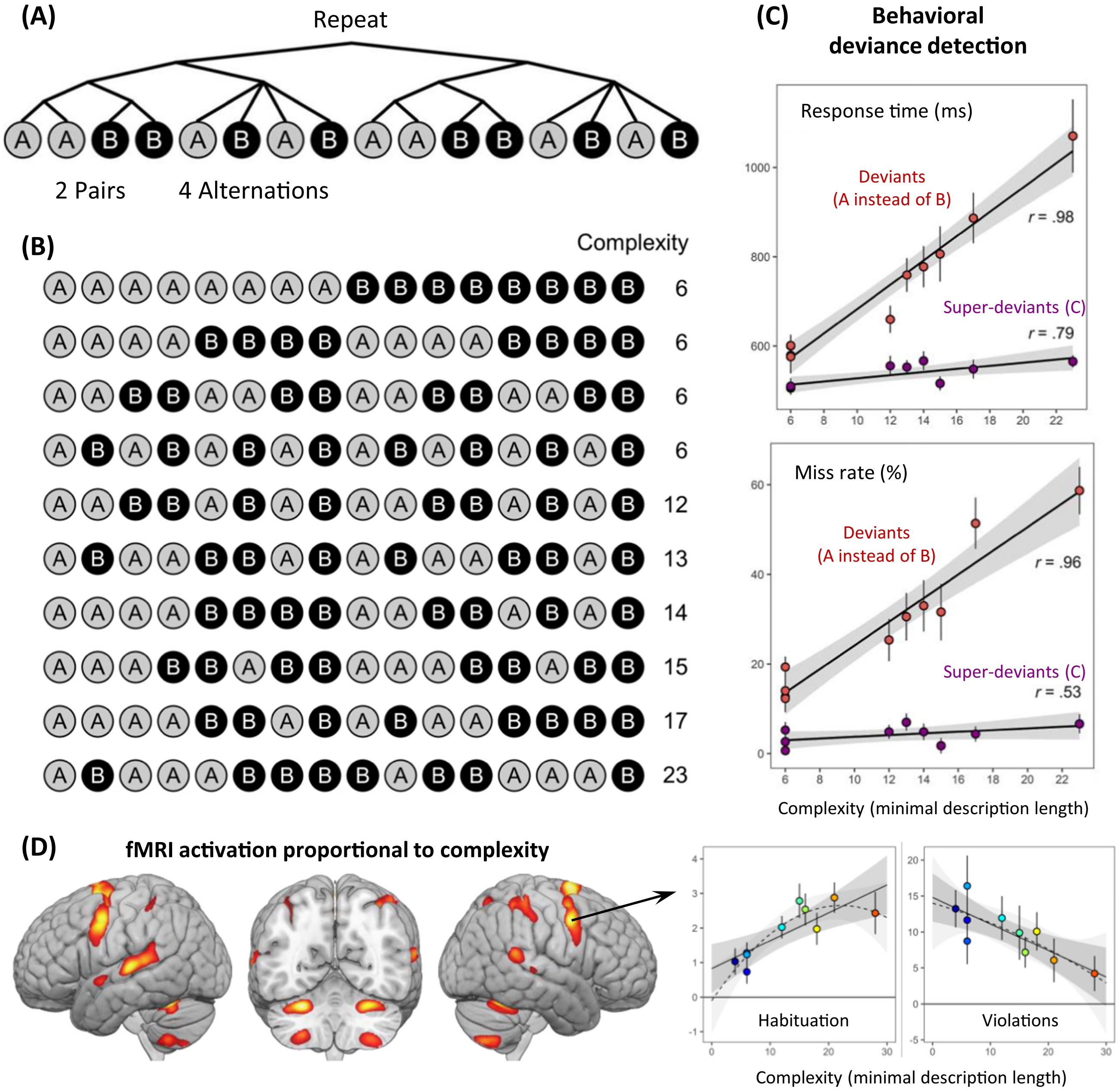

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Figure 4. A language for the human perception of binary sequences. (A) A mid-complexity sequence made of two sounds (A,B), which can be structured as a repetition of a concatenation of: (i) a repetition of repetitions (two pairs), and (ii) a repetition of four alternations. (B) We presented sequences of length 16, yet varying in their predicted complexity [i.e., minimum description length (MDL) in the proposed language]. (C) Behavioral evidence: the speed and accuracy with which human adults detect a deviant sound within such sequences is monotonically related to MDL [45]. (D) Functional fMRI evidence: while participants passively listen to a given sequence, brain activity increases monotonically with MDL in a bilateral premotor-temporal-cerebellar network. Furthermore, the reaction of this network to deviants is inversely proportional to MDL.

We then used fMRI and MEG to test the proposed language encoding at the brain level (S. Planton, F. Al Roumi *et al.,* unpublished). Assuming, within the predictive coding framework, that the brain uses the proposed language as an internal model to predict the upcoming items, we are led to two simple predictions:

(i) Brain signals related to model encoding should increase with sequence complexity (MDL).
(ii) Brain signals related to prediction error (evoked by deviant sounds) should decrease with MDL, as these sounds are predicted increasingly less well as complexity increases.

Both predictions were confirmed in fMRI and MEG. MRI located the effects on a network of bilateral areas, including auditory cortex (bilateral superior temporal gyri), premotor (bordering on dorsal BA 44), and anterior intraparietal regions (Figure 4D). The latter two regions overlapped with the regions previously found to be involved in the language of geometry and also intersected with a subject-specific localizer for basic arithmetic, thus suggesting their putative contribution to the repetition function, but this time coupled with superior temporal areas involved in sound representation. Again, there was only a minimal amount of overlap with subject-specific brain areas sensitive to natural language, in left BA 44 and left posterior superior temporal cortex. Those findings with elementary tone sequences therefore concur with the broader finding that music and language call upon largely distinct, dissociable brain circuits [46,47].

A proto-mathematical language for geometrical shapes

The two previous domains involved sequences (visuospatial or auditory) and thus resembled spoken language in this respect. We next wondered if a symbolic language of thought would also be needed to account for static geometric shapes. Two tests of this idea were developed.

First, we created a static intruder test, inspired by [48], in which participants had to detect a deviant shape among five repetitions of the same base shape [49]. For instance, the base shape could be a rectangle (with variations in size and orientation), and the deviant the same rectangle with the bottom right corner displaced (Figure 5A). Eleven quadrilateral shapes were tested. Although great care was taken to apply quantitatively identical amounts of deviancy, a large geometric regularity effect emerged: intruder detection was much easier when the base shape possessed geometric regularities (parallel sides, equal sides, right angles, or equal angles) than for irregular quadrilaterals. The effect was driven by shape complexity, which could be estimated by a simple count of these symbolic properties (Figure 5B). Thus, squares, rectangles, trapezoids, or parallelograms, which possess compressible regularities, are much easier to encode than random quadrilaterals devoid of those properties. A series of experiments demonstrated that this geometric regularity effect was highly replicable [49], for instance, in a classical serial search task, or even when the corner locations were presented in a sequence, thus bridging with our previous sequence work [23,24,35]. Importantly, the effect was replicated in preschoolers and in adults with little or no formal education (the Himba from Namibia), suggesting its independence from education. Undergoing fMRI studies suggest that the effect may originate from right anterior intraparietal cortex.

Second, we attempted to generalize this work to shapes beyond quadrilaterals [50]. We searched for an improved language of geometry that could account for the shapes attested in cultures throughout the world, including zig-zags, but also circles, spirals, arrays, friezes, etc. (Figure 1). We found that a minimal generalization of our original language [23] sufficed. Its primitives are again lines and numbers, concatenation, repetition and their recursive embedding, but also a capacity for continuous integration of curves. The latter is merely a generalization of the central concept of repetition with variation, but now extended to continuous tracing operations and capable of expressing, for instance, ‘keep tracing a curve while turning by a fixed amount’ (circle), or ‘keep tracing while turning and accelerating continuously’ (spiral). The simplest programs in this language generate a variety of plausible, human-like geometrical symbols.

According to our proposal, perceiving and drawing are tightly linked: when humans perceive a geometric shape and understand its structure, it means that they infer a short program capable

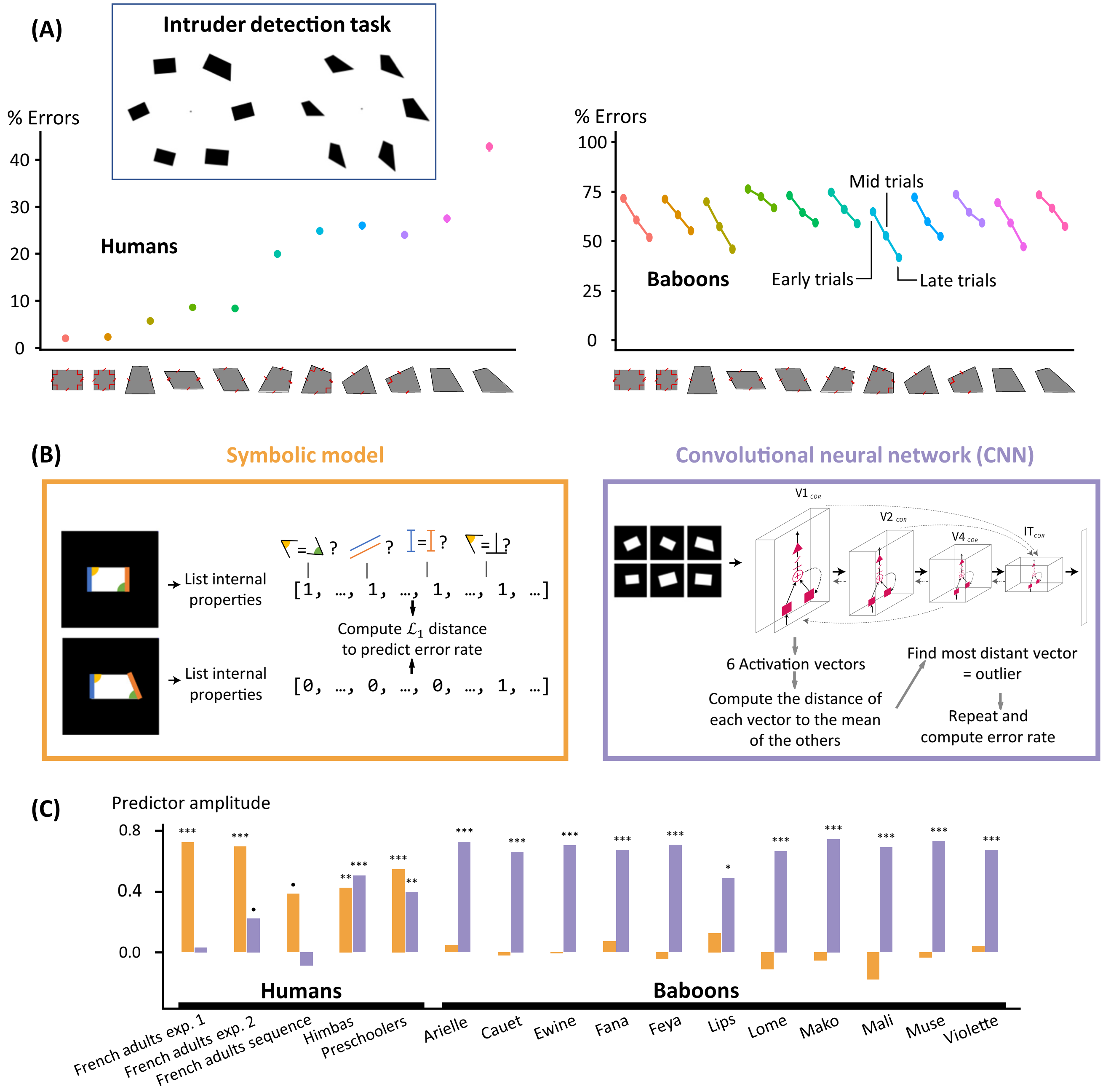

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Figure 5. Evidence for a human-specific sensibility to geometric shape. (A) Humans and baboons were trained to detect an intruder shape in a set of six. Five were identical quadrilaterals (up to a rotation and size change) while the intruder was defined by the displacement of one corner by a fixed amount. Humans (left), but not baboons (right), were highly sensitive to geometrical regularity and performed significantly better with shapes comprising symmetry, parallelism, right angles, or equal sides. Baboons improved across time, yet showed no sign of a geometrical regularity effect. (B) Two models for intruder detection: digital list of discrete geometric properties (symbolic model, left) versus analog vector-based image processing [convolutional neural network (CNN), right]. (C) Multiple regression showing that the CNN accounted for baboon behavior, while the symbolic model was needed in all human groups. Humans with no formal education (French preschoolers and Himba adults) were captured by a mixture of both regressors.

of generating it. We showed that such 'program induction' is feasible using the DreamCoder algorithm, a recent advance in computer science that allows more efficient search of the vast space of programs [51], and then performed two experiments to test it [50]. In the first experiment, we again showed that, in a match-to-sample working memory task, the difficulty of encoding and remembering a shape is determined by its MDL in our language. In a second experiment, we tested a more general prediction of our framework: shape complexity, if determined by MDL, should obey universal additive laws for concatenation, repetition, or embedding, valid in any proposition for a language of thought. For instance, any program for 'a circle of squares' should have a complexity predicted by the sum of the MDLs for circle, square, and embedding (this example nicely illustrates how compact programs can generate rich displays). Experimentally, we showed that human encoding and choice times respect these laws [50].

Non-human primates fail to grasp those languages

The present work is inspired by prior proposals of computer-like languages for shapes [52,53], visual and sequential patterns [54–56], and rhythmic musical sequences [57,58]. Its originality, however, lies in bringing this hypothesis down to simple tests that can be passed by children or adults without formal education and, most interestingly, by non-human primates. In this manner, we can evaluate the hypothesis that compositional languages are a prerogative of humans [13–15]. The shape intruder test, for instance, can be run identically in human and non-human primates: a monkey can easily learn to use a touch screen and touch the shape, among six, that differs from the others. With Joël Fagot, we trained 20 baboons to perform this task with a variety of nongeometric shapes (flowers, Gabor or color patches, etc.), and then transferred them to quadrilaterals. Would they, too, show a geometric regularity effect?

The results were clear-cut: baboons were insensitive to geometric regularity, both immediately upon transfer and after extensive training (Figure 5A). Unlike humans, they processed squares and rectangles no differently from other irregular quadrilaterals and the symbolic model did not capture any variance in their responses. Instead, their behavior was well-predicted by classical convolutional network models of the ventral visual pathway (Figure 5B,C), which conversely, were largely inoperative in educated humans. Interestingly, a mixture of the two models provided the best account of the performance of preschoolers and uneducated adults. Thus, two strategies are available to solve the geometric intruder task: a perceptual strategy, available to all primates, in which geometric shapes are processed within the ventral visual system as any picture or face would; and a symbolic strategy, seemingly available only to humans, whereby geometric shapes are compressed according to the discrete, symbolic 'repetitions of repetitions' (symmetries).

For the languages of visuospatial and binary auditory sequences, non-human primate testing is underway. The current results suggest that macaque monkeys fail to grasp even the simplest sequential patterns. Unlike preschoolers, monkeys were unable to memorize visuospatial sequences of eight locations on the octagon, so the test had to be restricted to sequences of four locations or less, which monkeys eventually managed, after thousands of training trials, to repeat forward or backward [59]. Even then, a direct comparison of adult monkeys with human preschoolers and adults showed striking behavioral differences [60]. All humans, regardless of age, were strongly sensitive to the overall sequence pattern; for instance, the simplest sequence, which humans found much easier to remember, was when the four locations were consecutive in a circle. Monkeys were totally insensitive to such patterns: they did not care about the transitions between locations and their performance varied with idiosyncratic changes in the starting point [60]. Single-cell recordings in monkey prefrontal cortex [61] confirm that monkey working memory is simply organized according to slots for each ordinal position, without any consideration of their relative geometric configuration.

Similarly, the 16-sound auditory sequences illustrated in Figure 4 probably lie beyond the limits of monkey working memory. Habituating monkeys and humans to a much simpler sequence of four sounds, such as AAAB, already revealed a sharp difference [62]. While both species could detect deviant sequences where the number was wrong (e.g., AAAAAB) or the last item was different (e.g., AAAA), only humans integrated those two properties into a unified representation of the entire sequence. This human-specific integration relied on bilateral human inferior frontal gyri and superior temporal sulci. Those findings and many others are consistent with the hypothesis that only humans grasp recursive languages [13–16,63]. There are a few dissenting data in macaques [64,65], however, so more extensive experiments, including in chimpanzees, will be needed to reach a firmer conclusion. It will also be important to test species other than non-human primates, such as vocal-learning birds, since it has been suggested that their behavior, brain circuits (at an abstract architectural level), and even genome resembles that of humans [66–70]. However, again, the bulk of the evidence suggests that their apparent success in learning complex sequential structures is not based on a genuine capacity for learning recursive languages [71,72].

Concluding remarks

We may now summarize the main aspects of our proposal.

(i) **Symbols, mental programs, and languages**. We propose that humans are characterized by a specific ability to attach discrete symbols to mental representations and to combine those symbols into nested recursive structures called mental programs, the compositional rules of which define a language of thought. Humans develop multiple such languages of thought in various domains (linguistic, musical, mathematical…).

(ii) **Conceptual productivity through compositionality**. Symbolic composition allows humans to create new concepts by recursive composition of existing ones (e.g., square = four-sided figure with equal sides and equal angles).

(iii) **Mental compression**. Understanding a sequence, a pattern, or a shape consists of compressing it into a compact mental object by inducing a short mental program capable of reproducing it.

(iv) **Complexity as MDL**. The psychological complexity of a concept is determined by the size of its mental program, its MDL in the appropriate language of thought [26–32].

(v) **Multiple languages**. In the human brain, distinct parallel circuits represent symbols and their recursive combinations in different domains. Within the domain of natural language, it was previously suggested that the inferior frontal region contains distinct parallel circuits for the compositional structures of phonology, morphosyntax, and semantics [19]; here we saw that even more dorsal areas of prefrontal and premotor cortex appear to be engaged in the languages of mathematics [40], visuospatial, and auditory sequences [24] (see also [47]). Other parallel circuits may exist for other recursive domains, for instance, theory of mind or planning [73].

(vi) **Mathematical regularity as repetition with variation**. The human perception of regularity rests primarily on a sensitivity to repetition, at multiple nested levels, possibly with variations (e.g., the parallel lines of a zig-zag or a square; the two pairs of an AABB sequence; etc.). Repetition with variation is essentially synonymous with the mathematical concept of symmetry (in mathematics, an object possesses a symmetry if it is invariant under some transformation, i.e., it repeats at an abstract level, over and above some lower-level variation). Indeed, the entire edifice of mathematics may be seen as a search for such regularities across number and space. The anterior intraparietal sulcus and prefrontal area 44d may constitute a core circuit

Outstanding questions

How are the proposed mental programs implemented in neuronal networks? To what extent do they rely on subcortical circuitry of the basal ganglia or cerebellum, in addition to the observed cortical areas? How do those circuits implement the basic operations of symbol assignment and recursive composition?

How can existing artificial neuronal networks be augmented to better mimic the discrete symbolic structures characteristic of human thought and the speed and ease of their acquisition from minimal data?

How early are the proposed languages of thought available during development? Infants are known to possess primitive concepts of number and geometry, but do they also possess, perhaps from birth, the recursive combinatorial faculty that allows them to combine these concepts and form compositive mathematical structures?

How do the known brain changes that accompanied hominization (brain size, prefrontal expansion, hemispheric asymmetries, long-distance connectivity, single-cell parameters, etc.) relate to the emergence of human abilities for symbols and languages?

What are the evolutionary precursors of human compositional ability? Do non-human primates possess, within a specific domain, a rudimentary combinatorial ability, for instance, in their ability to understand how objects or body parts attach to each other? Discovering such a domain would greatly facilitate the investigation of its neurophysiological mechanisms.

for recognition of repetition with variation, while connecting to other parietal or superior temporal areas, depending on the domain (e.g., visuospatial versus auditory-sequential).

(vii) **Human singularity**. Symbolic composition seems to be specifically human: non-human primates may learn to attach indexes to specific concepts, including quantities [11,74], but those signs do not seem to enter into compositional systems or grammars [16,63].

We readily acknowledge that these hypotheses require further empirical support and raise many questions (see Outstanding questions). For instance, do humans differ from other non-human primates since infancy, or is there a critical age at which compositional abilities emerge? Infants appear to possess early abilities for perceiving geometrical relationships [75], attaching symbols to concepts [76], and performing logical disjunctive inference [77,78], but whether they possess recursive programs has not yet been tested. The present paradigms may prove useful in this respect.

Also, how did the human faculty for languages evolve? Under the present framework, we expect evolutionary changes, not to a single area such as the left inferior frontal gyrus (Broca's area) [79], but rather to multiple, broad, and bilateral parallel circuits. Indeed, the differential expansion of association cortex [80–82] and long-range connectivity [83–86] in humans compared with all other primates suggest evolutionary changes in broad regions of inferior and middle prefrontal cortex together with higher-order associative parietal and temporal areas and their interconnections. Different circuits, however, do not necessarily imply different computations; on the contrary, we argue here that those circuits share a common trait of symbolic compositionality. An interesting possibility is that a similar mutation jointly granted compositionality to many, possibly all, human cortical circuits, for instance, through a change in laminar cortical organization, cell types, or dendritic tree computations [87–90]. Thus, multiple circuits would share a capacity for symbolic recursion while differing in their style and primitives (e.g., number, set and symmetry for math; merge and antisymmetry for language; pitch and rhythm for music; etc.) [19,58,91,92][i] (Figure 2).

Finally, a key outstanding question concerns how such a novel functionality is implemented in neural networks. Symbolic systems based on grammars go a long way towards explaining human inferences [51,52,93,94], but are hard to implement in neural hardware. We recognize that there are serious difficulties here, as well as empirical findings such as continuous prototypicality effects that seem best suited to a continuous neural network approach (for discussion, see e.g., [140]). A middle ground must be found. Indeed, the implementation of symbolic processing in artificial neural networks is a very active area of research, with massive progress in natural language comprehension and production [95–97], processing of complex mathematical expressions (e.g., [98–100]), and program inference using differentiable neural computing or 'neural Turing machines' [101]. In-depth analyses suggest that these networks contain compressed representations of the nested syntactic structures postulated by linguists [102,103]. However, their performance is often brittle and they often fail on basic tests of linguistic recursion [104–106], arithmetic [107], or geometry [49]. The present results support the currently unpopular view that discrete symbols and languages will play an essential role in any future model of the human mind. In support of this idea, internal communication through discrete symbols was recently found useful to improve state-of-the-art artificial neural networks [108]. Symbols may possess several advantages over continuous representations, such as robustness to noise, simplification, reduced bandpass, explicit communicability, etc. We suggest that, while many modular brain circuits may have kept their ancient evolutionary structure in the course of hominization, the human global neuronal workspace [109] became exponentially richer in humans due to symbolic composition and, therefore, correspondingly, a distinctly human neural code, supporting symbol processing, remains to be found.

Acknowledgments

This work was supported by INSERM (Institut National de la Santé et de la Recherche Médicale), CEA (Commissariat à l'Energie Atomique et aux Energies Alternatives), Collège de France, the Bettencourt-Schueller foundation, and a European Research Council ERC grant 'NeuroSyntax' to S.D. We gratefully acknowledge extensive discussions with Marie Amalric, Ghislaine Dehaene-Lambertz, Joël Fagot, Naama Friedmann, Christophe Pallier, Michael Posner, and Luigi Rizzi.

Declaration of interests

No interests are declared.

Resources

[i] http://ling.auf.net/lingbuzz/000959/v1.pdf

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